Influence of Temperature Change on Optimum Laminate Design

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Results of a laminate optimization study, which includes prescribed temperature change and mechanical loading conditions, are presented. Minimum-weight designs for balanced symmetric laminates are obtained subject to strength, membrane stiffness, and minimum thickness requirements, while including and omitting temperature change effects. Based on a first ply failure design philosophy and employing a linear thermoelastic analysis, it is shown that taking temperature decrease effects into account leads to substantial weight penalties in strength critical fiber composite laminates. Numerical results are presented for representative fiber composite materials based on three commonly used combined stress failure criteria.

Introduction

THE growing use of high-performance fiber composite materials in aerospace structural applications has stimulated interest in optimization procedures for the efficient design of laminates. The minimum-weight optimum design of laminates considering strength and stiffness constraints was studied first in the late 1960's by Foye[1] and Waddoups.[2] Both of these early efforts dealt directly with discrete numbers of plies, and they treated ply orientation angles as design variables. In order to improve computational efficiency and obtain more practical designs for thick laminates with many plies, recent studies (e.g., Refs. 3 and 4) have focused on treating material thickness t_i at each of several preassigned orientation angles θ_i as the operational set of continuous design variables. Although most of the previous reported work on optimum design of fiber composite laminates has considered multiple in-plane mechanical loading conditions, none of them have taken prescribed temperature change effects into account. However, a recently published study by Tsai and Hahn[5] points out that "curing stresses induced during fabrication can be very substantial so as to significantly decrease the first ply failure strength." Furthermore, Jones[6] has stated that "lamination theory including thermal effects is essential to the correct description of laminate behavior because of heterogeneity and the natural curing process for fabricating laminates."

The purpose of this paper is to report the results of a laminate optimization study in which prescribed temperature change and mechanical loading conditions are included. Adopting a first ply failure design philosophy, employing a linear thermoelastic analysis, and assuming a positive nonzero minimum thickness requirement for the material placed at each preassigned orientation angle, minimum-weight optimum laminate designs have been generated including and omitting a 200°F temperature decrease.

The 200°F temperature decrease is based on the assumption that the fabricated laminates are stress-free at 170°F, but that they are being used at an operating temperature of −30°F. Note that −30°F corresponds to the steady-state adiabatic wall temperature for Mach 0.7 at an altitude of 40,000 ft.

Results have been obtained for representative boron/epoxy and graphite/epoxy fiber composite materials, employing three commonly used combined stress failure criteria. It is found that the additional stresses induced by a 200°F temperature decrease lead to weight penalties of approximately 50 to 100%.

Problem Statement and Analysis

Consider a balanced symmetric laminate subject to multiple in-plane loading conditions $(N_{xk}, N_{yk}, N_{xyk})$ and prescribed uniform temperature changes ΔT_k (see Fig. 1). A minimum-weight design is sought subject to strength, membrane stiffness, and minimum thickness requirements. The material properties and the available orientation angles θ_i are understood to be preassigned parameters, and the thicknesses of materials t_i at each orientation angle θ_i are treated as continuous design variables. Design variable linking is employed to impose the balanced laminate requirement. For example, if the total number of orientation angles is $I=4$ and the θ_i are preassigned as $\theta_1=0°$, $\theta_2=+45°$, $\theta_3=-45°$, and $\theta_4=90°$, then the design variable linking $t_2=t_3$ imposes the balanced laminate requirement, and the number of independent thicknesses is reduced to three. Note that, for a heavily loaded structure involving thick laminates with many plies, the adjustments necessary to convert continuous variable optimum designs into roughly equivalent balanced and symmetric laminates should be straightforward.

The design optimization problem is to find the set of t_i, $i=1,2,\ldots I$, such that

$$W=\sum_{i=1}^{I} \rho_i t_i \tag{1}$$

subject to lower limits on the membrane stiffnesses

$$\frac{A_{11}}{A_{11}^{(L)}}-1\geq 0; \quad \frac{A_{22}}{A_{22}^{(L)}}-1\geq 0; \quad \frac{A_{66}}{A_{66}^{(L)}}-1\geq 0 \tag{2}$$

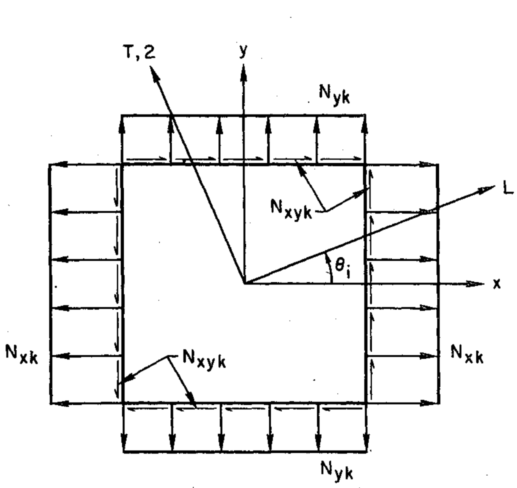

Fig. 1 Schematic of balanced symmetric laminate.

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Table 1 Applied load conditions

Load conditions k	N_{xk}, lb/in.	N_{yk}, lb/in.	N_{xyk}, lb/in.
1	12,000	3,000	6,000
2	−9,000	3,000	−6,000
3	6,000	3,000	9,000

and strength constraints based on one of the following three commonly used combined stress failure criteria:

Maximum Strain

$$1-Q_{jik}\geq 0;\quad j=1,2,\ldots J=6 \tag{3}$$

where

$$Q_{jik}=A_j^{(i)}\left(\frac{\sigma_{1ik}}{E_{Li}}-\nu_{TLi}\frac{\sigma_{2ik}}{E_{Ti}}\right)+B_j^{(i)}\left(\frac{\sigma_{2ik}}{E_{Ti}}-\nu_{LTi}\frac{\sigma_{1ik}}{E_{Li}}\right)+C_j^{(i)}\left(\frac{\tau_{12ik}}{G_{LTi}}\right);\quad j=1,2,\ldots J=6 \tag{4}$$

and the constants $A_j^{(i)}$, $B_j^{(i)}$, and $C_j^{(i)}$ are given in terms of allowable strains by Table I of Ref. 3.

Stress Interaction Relations

$$1-\frac{\sigma_{1ik}}{F_{Li}}\geq 0 \tag{5}$$

$$1-\left\{\left(\frac{\sigma_{2ik}}{F_{Ti}}\right)^2+\left(\frac{\tau_{12ik}}{F_{LTi}}\right)^2\right\}\geq 0 \tag{6}$$

Hill-Tsai

$$1-\left(\frac{\sigma_{1ik}}{F_{Li}^2}-\frac{\sigma_{1ik}\sigma_{2ik}}{F_{Li}F_{Ti}}+\frac{\sigma_{2ik}^2}{F_{Ti}^2}+\frac{\tau_{12ik}^2}{F_{LTi}^2}\right)\geq 0 \tag{7}$$

and subject to minimum thickness limitations

$$t_i\geq t_i^{(L)} \tag{8}$$

for $i=1,2,\ldots I$ and $k=1,2,\ldots K$. It is emphasized that the strength failure criteria are applied at the ply level for all orientation angles I and all load conditions K. In order to facilitate comparison with the other failure criteria [Eqs. (5-7)], the maximum strain failure envelope [Eqs. (3) and (4)] is represented by a set of J planar facets in $(\sigma_1,\sigma_2,\tau_{12})$ stress space rather than in $(\epsilon_1,\epsilon_2,\gamma_{12})$ strain space. The allowable stress notation F_L, F_T, and F_{LT}, used in the interaction formulas [Eqs. (5) and (6)] and the Tsai-Hill constraint [Eq. (7)], are drawn from Ref. 7, and it provides for different allowable stresses in tension and comparison.

Since attention is restricted to symmetric laminates, no elastic coupling between membrane and bending behavior exists. Furthermore, since this study is limited to laminates that are balanced as well as symmetric, it follows that the laminate membrane behavior is grossly orthotropic with respect to the x,y reference coordinate system shown in Fig. 1 (i.e., the membrane stiffnesses $A_{16}=A_{26}=A_{61}=A_{62}=0$). Assuming plane stress and adopting the Kirchoff-Love deformation hypothesis, the basic force-deformation relations governing the behavior of the symmetric balanced laminates subject to in-plane mechanical loading and uniform temperature change conditions are

$$N_{xk}=A_{11}\epsilon_{xk}+A_{12}\epsilon_{yk}-\bar{A}_1\Delta T_k \tag{9a}$$

$$N_{yk}=A_{21}\epsilon_{xk}+A_{22}\epsilon_{yk}-\bar{A}_2\Delta T_k \tag{9b}$$

$$N_{xyk}=A_{66}\gamma_{xyk} \tag{9c}$$

where the stiffnesses are given by

$$A_{rs}=\sum_{i=1}^{I}(C'_{rs})_i t_i;\quad r,s=1,2,6 \tag{10}$$

the coefficients (C'_{rs}) depend upon the elastic properties (C_{rs}) of the lamina materials and the orientation angles (θ_i) (see Ref. 3, Appendix II). The coefficients $\bar{A}_1$ and $\bar{A}_2$ can be expressed as follows:

$$\bar{A}_1=\sum_{i=1}^{I}\left\{\left[(C_{11})_i\ell_i^2+(C_{12})_i m_i^2\right]\alpha_{1i}t_i+\left[(C_{12})_i\ell_i^2+(C_{22})_i m_i^2\right]\alpha_{2i}t_i\right\} \tag{11}$$

$$\bar{A}_2=\sum_{i=1}^{I}\left\{\left[(C_{11})_i m_i^2+(C_{12})_i\ell_i^2\right]\alpha_{1i}t_i+\left[(C_{12})_i m_i^2+(C_{22})_i\ell_i^2\right]\alpha_{2i}t_i\right\} \tag{12}$$

where α_{1i} and α_{2i} represent the longitudinal and transverse thermal expansion coefficients, respectively, of the material placed at the θ_i orientation. Furthermore it is understood that $\ell_i=\cos\theta_i$ and $m_i=\sin\theta_i$. It is important to note that the thermal force terms in Eq. (9) (i.e., $A_1\Delta T_k$ and $A_2\ \Delta T_k$) are linearly dependent on the thicknesses t_i, whereas the mechanical loading (N_{xk},N_{yk},N_{xyk}) is independent of the thicknesses t_i. This implies that, although high stresses induced by mechanical loads generally can be alleviated by increasing material thicknesses, the same is not true with respect to high stresses induced by temperature changes.

For any particular laminate design, it is easy to solve Eq. (9) for the laminate strains ϵ_{xk}, ϵ_{yk}, and γ_{xyk}. The strains in the lamina are determined using the following transformation relations:

$$\epsilon_{1ik}=\ell_i^2\epsilon_{xk}+m_i^2\epsilon_{yk}+m_i\ell_i\gamma_{xyk} \tag{13a}$$

$$\epsilon_{2ik}=m_i^2\epsilon_{xk}+\ell_i^2\epsilon_{yk}-m_i\ell_i\gamma_{xyk} \tag{13b}$$

$$\gamma_{12ik}=2m_i\ell_i\epsilon_{xk}+2m_i\ell_i\epsilon_{yk}+(\ell_i^2-m_i^2)\gamma_{xyk} \tag{13c}$$

and the stresses at the ply level are given by

$$\sigma_{1ik}=(C_{11})_i\epsilon_{1ik}+(C_{12})_i\epsilon_{2ik}-[(C_{11})_i\alpha_{1i}+(C_{12})_i\alpha_{2i}]\Delta T_k \tag{14a}$$

$$\sigma_{2ik}=(C_{12})_i\epsilon_{1ik}+(C_{22})_i\epsilon_{2ik}-[(C_{12})_i\alpha_{1i}+(C_{22})_i\alpha_{2i}]\Delta T_k \tag{14b}$$

$$\tau_{12ik}=(C_{66})_i\gamma_{12ik} \tag{14c}$$

Optimization Procedure

The optimization procedure employed in this study is based on the quadratic extended interior penalty function set forth in Ref. 8. This formulation leads to a sequence of unconstrained minimization problems, each of which is solved using the modified Newton method presented in Ref. 9. The transition parameter ϵ, which plays such a central role in the extended penalty function definition, is determined using the formulas developed in Ref. 10. The constraint deletion technique given in Ref. 3 is used to overcome the difficulties associated with the large number of constraint functions involved in the strength requirements. Only critical and potentially critical constraints are retained, and the set to be retained is updated at the beginning of each unconstrained minimization stage. Also, as previously mentioned, design variable linking is used to impose gross orthotropy on the laminate, thus protecting the validity of the balanced laminate analysis.

Numerical Examples

Numerical results that demonstrate the influence of a modest temperature decrease on the minimum-weight optimum design of fiber composite laminates are presented. The example problem treated involves three distinct load conditions, and they are stipulated in Table 1. Minimum-weight laminate designs are sought for four commonly used fiber composite materials using each of the three strength failure criteria options previously discussed [see Eqs. (3-7)]. The laminate in each example is assumed to be made from a single kind of fiber composite material. The material properties used were drawn from Ref. 11, and they are summarized in Table 2. For all 12 combinations of material and strength failure

Table 2 Unidirectional material properties

Material	E_L, psi ($\times 10^6$)	E_T, psi ($\times 10^6$)	G_{LT}, psi ($\times 10^6$)	ν_{LT}	ρ, lb/in.^{3}	α_l, in./in./°F ($\times 10^{-6}$)	α_2, in./in./°F ($\times 10^{-6}$)	F^t_L, ksi	F^c_L, ksi	F^t_T, ksi	F^c_T, ksi	F^s_{LT}, ksi
Boron epoxy	30	2.7	0.7	0.21	0.0725	2.3	10.6	192	−353	10.4	−40	15.3
High-mod Gr/Ep	25	1.7	0.65	0.3	0.056	−0.3	19.5	110	−100	4	−20	9
High-str Gr/Ep	21	1.7	0.65	0.21	0.056	−0.21	16.0	180	−180	8	−30	12
Inter-str Gr/Ep	17	1.7	0.65	0.21	0.055	0.3	16.0	160	−160	7.5	−25	10

Table 3 Influence of temperature decrease on minimum-weight laminate designs

Strength criterion	Available orientation i	θ_i	Boron/epoxy $\Delta T = 0$°F	Boron/epoxy $\Delta T = -200$°F	High-mod Gr/Ep $\Delta T = 0$°F	High-mod Gr/Ep $\Delta T = -200$°F
Maximum strain criterion	1	0°	22.72[a]	26.22	22.16	...
	2, 3	±45°	70.84	70.50	71.40	...
	4	90°	6.44	3.28	6.44	...
		t_{tot},	0.2588	0.4266	0.5242	...
		W/in.^{2}	0.0191	0.0309	0.0294	...
Stress interaction formula	1	0°	23.45	29.17	26.52	...
	2,3	±45°	73.70	70.24	71.59	...
	4	90°	2.85	0.59	1.99	...
		t_{tot},	0.249	0.3738	0.444	...
		W/in.^{2}	0.0181	0.0271	0.0249	...
Hill-Tsai criterion	1	0°	23.07	28.43	23.52	...
	2, 3	±45°	72.64	70.22	69.87	...
	4	90°	4.29	1.35	6.61	...
		t_{tot},	0.2522	0.381	0.4817	...
		W/in.^{2}	0.0183	0.0281	0.0269	...

[a] All material distributions are given as % t_{tot}.

Table 4 Influence of temperature decrease on minimum-weight laminate designs

Strength criterion	Available orientation i	θ_i	High-str Gr/Ep $\Delta T = 0$°F	High-str Gr/Ep $\Delta T = -200$°F	Inter-str Gr/Ep $\Delta T = 0$°F	Inter-str Gr/Ep $\Delta T = -200$°F
Maximum strain criterion	1	0°	18.14[a]	32.33	19.46	31.11
	2, 3	45°	77.10	67.66	76.66	68.88
	4	90°	4.76	0.01	3.88	0.01
		t_{tot},	0.3408	0.8049	0.4151	0.9073
		W/in.^{2}	0.0191	0.0451	0.0228	0.0494
Stress interaction formula	1	0°	18.08	33.00	19.85	32.61
	2, 3	45°	80.66	67.98	79.98	67.38
	4	90°	1.26	0.02	0.17	0.01
		t_{tot},	0.328	0.6982	0.40	0.7865
		W/in.^{2}	0.0184	0.0391	0.0221	0.0431
Hill-Tsai criterion	1	0°	17.02	32.23	19.07	31.75
	2, 3	45°	77.26	67.76	77.50	68.24
	4	90°	5.72	0.01	3.43	0.01
		t_{tot},	0.340	0.725	0.415	0.810
		W/in.^{2}	0.0191	0.041	0.0226	0.044

[a] All material distributions are given as % t_{tot}.

criteria, minimum-weight optimum designs are sought, first excluding and then including the influence of a 200°F temperature decrease. It is assumed that the material properties are constant over the temperature range from +170° to −30°F. In all of these examples, the minimum membrane stiffnesses were $A_{11}^{(L)} = 2.25 \times 10^6$ lb/in., $A_{22}^{(L)} = 1.5 \times 10^6$ lb/in., and $A_{66}^{(L)} = 1.5 \times 10^6$ lb/in., whereas the minimum thickness limitation was $t_i^{(L)} = 0$. Attention was limited to 0°, ±45°, 90° laminates that are balanced and symmetric; therefore each of the 24 laminate design optimization problems involves only three independent design variables. The results obtained for all 24 examples are displayed in Tables 3 and 4. For each example, the percentage of the total thickness to be placed at 0°, ±45°, and 90° is given, along with the total laminate thickness t_{tot} and the weight per unit area of laminate surface. Table 3 shows the results for boron/epoxy (B/Ep) and high-modulus graphite-epoxy material (high-mod Gr/Ep). The B/Ep results show a 50% weight increase when the 200°F temperature decrease effect is taken into account, independent of the strength failure criterion option employed. In the case of the high-mod Gr/Ep material, it is not possible to find a feasible design when the 200°F temperature decrease is taken into account. This is not surprising in view of the observation that, for high-mod Gr/Ep (see Table 2),

$$-\alpha_T E_T \Delta T = 6630 > F_T^t = 4000 \text{ psi}$$

Table 4 displays the results for high-strength graphite-epoxy (high-str Gr/Ep) and intermediate-strength graphite-epoxy (inter-str Gr/Ep) material. For both of these materials, the results show a 100% weight increase when the 200°F temperature decrease effect is taken into account, independent of the strength failure criterion option selected. It also is interesting to note that, when the temperature change is not included, the minimum weights achieved with the B/Ep and the high-str Gr/Ep are substantially the same. However, when the 200°F temperature decrease is included, the minimum-weight designs obtained using the high-str Gr/Ep material are 50% heavier than those achieved with the B/Ep material.

Conclusions

The results reported here indicate that temperature decreases can produce significant weight penalties in strength critical fiber composite laminates. The basic cause of this weight penalty is the mismatch in thermoelastic properties between the various lamina making up the laminate. Although the simple linear elastic stress analysis used here tends to overestimate thermal stresses, the 200°F temperature decrease employed is modest. Furthermore, some recently reported results[12] indicate that thermally induced residual stresses are often about 80-100% of those predicted by linear analysis.

It should be recognized clearly that the findings presented here are predicted on a design philosophy that seeks to prevent first ply failure, even if it is a transverse tension failure mode. The importance of this point is underscored by the fact that, when temperature decrease is included, almost all of the critical strength constraints involve the transverse tension failure mode in various lamina. It also is interesting to note that, in the examples without temperature change, the membrane shear stiffness constraint is critical, whereas for those with the 200°F temperature decrease none of the membrane stiffness constraints are critical.

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